# Three-Dimensional Heat-and Mass-Transfer Effects across High-Speed Reattaching Flows

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**A theoretical study is made of periodic spanwise disturbances in nominally two-dimensional reattaching laminar and turbulent separated flows. A compressible small disturbance flow analysis of the local vortex instability mechanism involved is made emphasizing the three-dimensional heat-transfer effects including blowing or suction through the surface. It is shown that Reynolds analogy does not apply between the disturbance skin friction and heat transfer. The corresponding thermal response of the wall surface is also analyzed taking into account the spanwise heat conduction within the underlying surface materials; it is governed by the characteristic ratio of boundary layer heat transfer to the spanwise heat conduction. The theoretical predictions are in good agreement with experimental observations.**

## Nomenclature

$B$ = stagnation point inviscid velocity gradient $(du_e/dx)_s$
$C_p$ = constant pressure specific heat
$f$ = similarity solution stream function
$g$ = total enthalpy ratio, $H/H_s$
$h,H$ = static and total enthalpy, respectively $[H=h+(u^2/2)]$
$k$ = thermal conductivity
$M$ = Mach number
$p$ = static pressure
$\dot{q}$ = heat-transfer rate
$T$ = absolute static temperature
$u,v,w$ = streamwise, normal and sidewash velocity components in $x,y,z$ directions, respectively
$x,y,z$ = streamwise coordinates measured from reattachment line, distance normal to surface and spanwise distance, respectively
$Y$ = compressibly-transformed $y$ $(dY=\rho_0 dy)$
$\alpha$ = inverse wavelength parameter $(2\pi/\lambda)$
$\bar{\alpha}$ = $\alpha\sqrt{\nu_{0_s}/B}$
$\delta$ = boundary-layer thickness
$\eta$ = similarity coordinate = $Y\sqrt{B/\rho_w\mu_w}$
$\lambda$ = spanwise wavelength of disturbance pattern
$\mu$ = coefficient of viscosity
$\theta$ = nondimensional wall temperature function [Eq. (11)]
$\nu$ = kinematic viscosity $(\mu/\rho)$
$\rho$ = density
$\tau$ = shear stress

*Subscripts*

$e$ = effective edge of boundary layer
$g$ = gas side of wall
$1$ = perturbation values in first approximation
$0$ = basic two-dimensional reattaching flow
$s$ = stagnation or reattachment line conditions
$w$ = wall surface

## I. Introduction

THE study of separated flow reattachment is of basic importance in many fluid-mechanics and heat-transfer problems. In recent years, an appreciable body of evidence has accumulated[1-9] showing that significant 3-D effects with vortex-like laterally periodic variations in shear, pressure, and heat transfer may occur in nominally two-dimensional reattaching separated laminar and turbulent flows, under both low- and high-speed flow conditions. Figure 1 illustrates a typical reattachment pattern observed downstream of a rearward-facing step in supersonic flow.[5] Such disturbances exhibit the following properties over a wide range of Mach and Reynolds numbers in both two-dimensional and axisymmetric reattaching flows, regardless of the particular upstream cause of separation (backward facing steps, compression ramp, etc.). 1) The spanwise wave length $\lambda$ is comparable to the Taylor-Görtler value of 2 to 4 boundary-layer thicknesses. 2) They have a definite vortex-like structure, consisting of a layer of alternating vortex pairs located near the boundary-layer edge with a maximum sidewash around the center of the layer and rapid damping near the wall. 3) Pronounced lateral peaks (10-25% of mean) in pitot pressure, skin friction, and heat transfer occur which are strongly correlated with each other and the spanwise vortex location and which increase markedly with Mach number. 4) Provided there is some kind of upstream streamwise vorticity disturbance, however weak and irregular, the three-dimensional disturbances assume a charactertistic well-organized form only following the occurence of reattachment.

A previous detailed investigation[10] has shown that these observed features are likely associated with the birth of vortices during reattachment due to a local disturbance mode unique to this type of flow (i.e., a Taylor-Görtler-type of

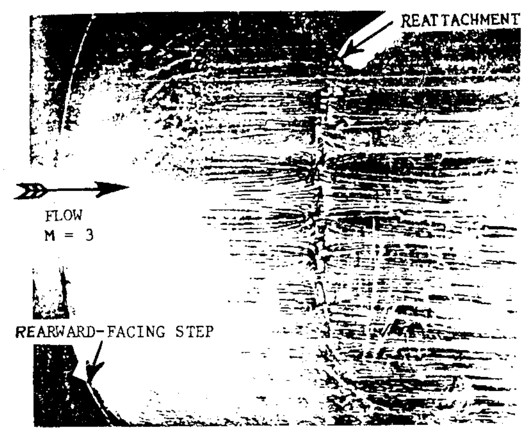


**Fig. 1 Reattachment disturbance pattern (surface oil film[5]).**

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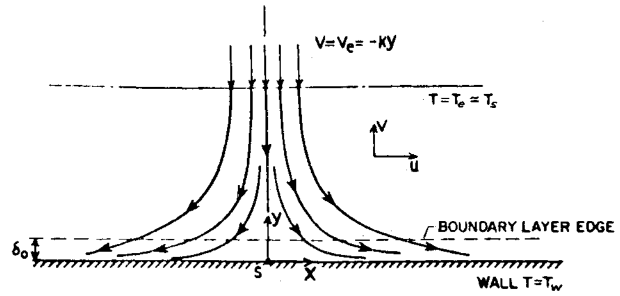


**Fig. 2 Idealized 2-D reattaching flow model.**

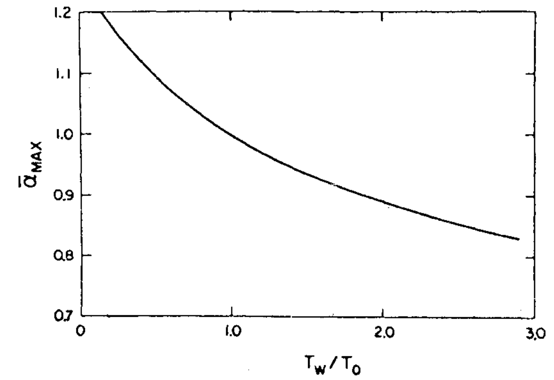


**Fig. 3 Maximum nondimensional vortex disturbance wavelength parameter vs wall temperature ratio.**

instability associated with the interaction between the concave streamline curvature of the turning stagnation flow and the boundary-layer profile) wherein faint random incoming disturbances merely act as an initial "trigger" and have no significant effect on the final steady-state behavior. In the present paper, the heat-transfer aspects of this disturbance mechanism are examined, including the thermal coupling between the 3-D external boundary-layer disturbances and the heat conduction within the underlying surface material. This is important because it determines the actual temperature response of the wall surface which is observed in practice. In order to serve possible future applications where blowing or suction occurs within the reattachment region, the vortex disturbance theory also is extended to include surface mass transfer in the basic flow.

## II. Theoretical Analysis

### A. Assumptions

Since the analysis of even two-dimensional reattaching high-speed flows is a difficult problem, the following simplifying assumptions are introduced to enable a first-approximation treatment of the 3-D effects. 1) The flow consists of a basic nonparallel two-dimensional incoming stream plus small linearized 3-D steady-state perturbations. The amplitude solutions of the resulting disturbance equations are assumed to approximate closely the actual neutrally stable disturbance values that emerge from the prior transient development. 2) Attention is confined to a small neighborhood of the reattachment line and approximates the incoming basic flow outside the boundary layer as an infinitely wide uniform inviscid flow undergoing a stagnation reattachment normal to the surface (Fig. 2). 3) Under the presumed high Reynolds numer flow conditions of interest, the viscous and heat-conduction effects on the disturbance field are treated by boundary-layer-type approximations. 4) The laminar density-viscosity product is constant and equal to its basic flow value ($\rho\mu \simeq \rho_0\mu_0 \simeq \rho_{w0}\mu_{w0}$) with a unit Prandtl number. 5) In the case of turbulent incoming flow, it is assumed that the effect of such turbulence is negligible as compared to that of laminar viscosity in the primary formative mechanism of the disturbance vortices near the stagnation point. This is substantiated by several pieces of experimental evidence.[11,12] 6) Any suction or blowing through the wall is prescribed, homogeneous (air-to-air), normal to the surface, and entails no phase change on the surface. 7) The thermal conductivity of the wall material is a constant.

### B. Disturbance Flow Equations

Considering the stagnation-point region, and introducing the compressible similarity variable $\eta$, the basic nonadiabatic flow is self-similar, and is described by $u_0 = Bxf_0'(\eta)$, $\rho_0 v_0 = -\sqrt{B\rho_w\mu_w}\, f_0(\eta)$, $H_0 = H_{0s}g_0(\eta)$, with $p_0 = p_{0s} - (\rho_{0_s}B^2x^2/2)$, where $f_0$ and $g_0$ are governed by the well-known momentum and energy equations.

$$f_0 f_0'' + f_0''' = (f_0')^2 - g_0 \tag{1}$$

$$f_0 g_0' + g_0'' = 0 \tag{2}$$

subject to the outer boundary conditions $f_0'(\infty) = g_0(\infty) = 1$; in addition, the wall conditions $f_0(0) = f_{0w}$, $f'(0) = 0$ and $g_0 = h_w/H_{0_s} \simeq T_w/T_{0_s}$, with a given wall temperature $T_w$ (here negative $f_{0w}$ implies blowing). The concave streamline curvature of this highly nonparallel flow is implicit in Eqs. (1) and (2).

The preceding assumptions simplify the compressible Navier-Stokes equations as applied to arbitrary steady-state small three-dimensional disturbances $u'(x,y,z) = u - u_0$, $v'(x,y,z) = v - v_0$, $w = w'(x,y,z)$, $p'(x,y,z) = p - p_0$, $H'(x,y,z) = H - H_0$ and $\rho'(x,y,z) = \rho - \rho_0$. Then imagine that a purely local instability has lead to a steady-state laterally periodic disturbance field, which in view of the highly nonparallel stagnation-like character of the basic flow is postulated to have the following functional form near the reattachment line (neglecting terms of order $x^2$):

$$u'(x,y,z) = Bxu_1(\eta)\cos\alpha z \tag{3a}$$

$$v'(x,y,z) = -\sqrt{B\nu_{0_s}}\, v_1(\eta)\cos\alpha z \tag{3b}$$

$$w'(x,y,z) = \alpha\nu_{0_s} w_1(\eta)\sin\alpha z \tag{3c}$$

$$p'(x,y,z) = \rho_{0_s}B\nu_{0_s}P_1(\eta)\cos\alpha z \tag{3d}$$

$$H'(x,y,z) = H_{0_s}g_1(\eta)\cos\alpha z \tag{3e}$$

where $\alpha = 2\pi/\lambda$. Then, substituting into the general small disturbance equations and taking into account the basic flow relations, the perturbation distribution functions $u_1$, $v_1$, etc., for a nonadiabatic compressible flow are found to be governed by the following linear ordinary differential equations:

$$u_1 - (v_1/g_0)' + \bar{\alpha}^2 w_1 \simeq -f_0(g_1/g_0)' \tag{4}$$

$$f_0 u_1' - 2f_0' u_1 + (v_1/g_0) f_0'' + u_1'' \simeq -g_1/g_0 \tag{5}$$

$$f_0 w_1' + w_1'' + g_0 P_1 \simeq 0 \tag{6}$$

$$f_0 g_1' + g_1'' + g_0'[(v_1/g_0) - (g_1 f_0/g_0)] \simeq 0 \tag{7}$$

$$\left(\frac{P_1'}{g_0}\right)' - \bar{\alpha}^2 g_0 P_1 - 2\frac{(f_0'/\sqrt{g_0})'\sqrt{g_0}P_1'}{g_0 f_0'} \simeq -2\frac{\bar{\alpha}^2 g_0}{f_0'} w_1 \tag{8}$$

where a prime denotes differentiation with respect to $\eta$. Equations (4-7) derive from continuity, streamwise momentum, spanwise momentum, and energy conservation, respectively. Equation (8) is an approximate form of the $y$-momentum equation[10] derived under the assumption [consistant with Eq. (3)] that the viscous effects on the disturbance pressure are negligable; note, however, that it is second order with respect to $\eta$. In addition, from the ideal gas equation of state in which pressure variations across the boundary layer are assumed to influence density negligibly compared to temperature changes, we have $\rho_1/\rho_{0_s} \simeq -g_1/g_0^2$.

To this order of approximation, only the $x$ component of disturbance vorticity $\zeta' = (\partial w'/\partial y) - (\partial v'/\partial z) = \bar{\alpha}B[g_0^{-1}(dW_1/d\eta) - V_1]\sin\alpha z$ is significant, so that the disturbance field involves vortices parallel to the wall which develop in the turning reattachment flow. Equations (4-8) apply to high-speed laminar or turbulent flow, and include compressibility and heat-transfer effects, plus an explicit equation for the normal pressure gradient. Note that the pressure and sidewash perturbations are intimately coupled, but are independent of the other disturbances in the sense that Eqs. (6) and (8) involve $w_1$ and $P_1$ only; once they are solved, Eqs. (4, 5, and 7) then determine $u_1$, $v_1$, and $g_1$.

Taking the wall temperature and mass transfer as given, the disturbance boundary conditions are $u_1(0) = w_1(0) = 0$ (no slip), $v_1(0) = g_1(0) = 0$. Furthermore, consistent with our neglect of viscous terms in Eqs. (8), we can apply the inviscid normal momentum condition that $dP_1/d\eta(0) = 0$. Far from the surface, since the disturbances are presumed to originate during the act of reattachment process itself and vanish upstream, we have $u_1(\infty) = w_1(\infty) = g_1(\infty) = P_1(\infty) = 0$; moreover, in order to insure that the farfield pressure solution of the second-order Eq. (8) behaves properly, we can impose the condition that its asymptotic behavior be such that $v_1(\infty) \to 0$. Thus, we have an eigenvalue problem where nontrivial disturbance solutions exist only for certain values of $\alpha$ as described in the following. Once these solutions are found, their gradients can be used to evaluate the streamwise and lateral wall shear stresses, respectively, as follows

$$\tau_{w,x} \simeq \mu_{0_w}\left(\frac{\partial u_0}{\partial y} + \frac{\partial u'}{\partial y}\right)_w \simeq \rho_{0_s}\sqrt{B^3}x\;[f_0''(0)\cos\alpha z] \tag{9a}$$

$$\tau_{w,z} \simeq \mu_{0_w}\left(\frac{\partial w'}{\partial y}\right)_w \simeq \rho_{0S} B\nu_{0_s}\bar{\alpha} w_1'(0)\sin\alpha z \tag{9b}$$

while the corresponding first-order approximation to the reattachment heat transfer is

$$q_w \simeq \rho_{0_s}\sqrt{\nu_{0_s}B}\,H_{0_s}\,[g_0'(0) + g_1'(0)\cos\alpha z] \tag{10}$$

### C. Thermal Response within Surface

The functional form of Eq. (3) suggests that the corresponding underlying surface material temperature disturbance is of the form

$$T'(x,y,z) = (H_{0_s}/C_p)\theta(y)\cos\alpha z + 0(x^4) \tag{11}$$

where $-\infty \leq y \leq 0$. Then substituting into the steady-state nonablating three-dimensional heat-conduction equation yields, for small $x$,

$$\frac{d^2\theta}{dy^2} - \alpha^2\theta = 0 \tag{12}$$

Far below the surface $y \to -\infty$, the temperature disturbance vanishes so that $\theta(-\infty) = 0$. Conservation of energy across the gas-solid interface $y = 0$ requires that the heat conduction away from the interface within the wall be equal to the external boundary-layer heat-transfer disturbance into the surface

$$k_w\left(\frac{\partial T'}{\partial y}\right)_w = \left(\frac{k}{C_p}\right)_{g,w}\left(\frac{\partial H'}{\partial y}\right)_{g,w} \tag{13}$$

which, upon substitution of Eqs. (3) and (11), yields

$$\frac{d\theta}{dy}(0) = \frac{k_{g,w}}{k_w}\left(\frac{d\eta}{dy}\right)_{g,w} g_1'(0) \tag{14}$$

where $(d\eta/dy)_{g,w} = \sqrt{B\rho_{0w}/\mu_{0w}}$ and $g_1'(0)$ is presumed to be known in the first approximation from the boundary-layer vortex enthalpy disturbance field solution in Sec. IIIA, obtained for a given $g_1(0) = 0$. It will be modified in the following to include the back-effect of the wall temperature perturbation.

The required solution of Eq. (12) is

$$\theta = \frac{k_{g,w}}{\alpha k_w}\left(\frac{d\eta}{dy}\right)_{g,w} g_1'(0)e^{\alpha y} \tag{15a}$$

with

$$\theta(0) = \frac{k_{g,w}(d\eta/dy)_{g,w}}{\alpha k_w} g_1'(0) \tag{15b}$$

This result shows that the surface temperature response to the heating effect of the disturbance vortices depends on the parameter

$$\kappa = \frac{k_{g,w}(d\eta/dy)_{g,w}}{\alpha k_w} \tag{16}$$

which characterizes the ratio of boundary-layer surface heat transfer to spanwise material heat conduction. For example, when $\kappa$ is very small (large relative spanwise heat-conduction effect), the surface temperature response vanishes owing to the smearing out of the external heat-transfer perturbation by heat conduction. On the other hand, when the external heat-transfer disturbance is of the same order or larger than spanwise heat conduction, $\theta(0)$ can be appreciable; this occurs not only for surface materials of low thermal conductivity under moderate external heat transfer, but also for high conductivity materials subject to the very large heating typical of hypersonic flows.[3]

Since $T_w'$ is unknown a priori, $g_1'(0)$ in Eq. (15) actually should be based on a small nonzero wall enthalpy perturbation $g_1(0) = C_p T_w'/H_{0_s}\cos\alpha z = \theta(0)$ given by continiuty of interface temperature in the absence of phase change on the surface. However, the back-effect of this $g_1(0) \neq 0$ on $g_1'(0)$ is estimated easily since, for small $\theta(0)$, it is linear and approximately proportional to the wall temperature effect on the basic 2-D flow heat transfer; hence,

$$g_1'(0) \simeq [1 - \theta(0)]\;[g_1'(0)]_{g_{1_w}=0} \tag{17}$$

from which Eq. (15b) now yields the solution

$$\theta(0) \cong \hat{\kappa}/1 + \hat{\kappa}) \tag{18}$$

where $\hat{\kappa} = \kappa[g_1'(0)]_{g_{1_w}=0}$. Equation (18) shows that the wall temperature response to the external vortex disturbance heating is bounded for all $\kappa$, ranging from a small value $\sim\hat{\kappa}$ when the surface conductivity is dominant to a limit of unity in the opposite extreme of external heat transfer dominated disturbances.

## III. Results

### A. Theory

Numerical solutions of Eqs. (4-8) were obtained with a standard Runge-Kutta integration routine, using a straightforward "shooting" method. Inviscid asymptotic solutions[10] for large $\eta$ were used to enforce the outer boundary conditions at a finite large value of $\eta$ (to which the results were insensitive). Owing to the assumption of infinite incoming reattaching flow width and lack of a body scale dimension, a continuum of eigensolutions is found within the band $0 \leq \bar{\alpha} \leq \bar{\alpha}_{max}$, where $\bar{\alpha}_{max}$ is unity for adiabatic walls and increases slightly with increasing wall cooling $(T_w/T_{s_0} < 1)$, as shown in Fig. 3. Our method of dealing with this feature in comparing with experiment is discussed in the following.

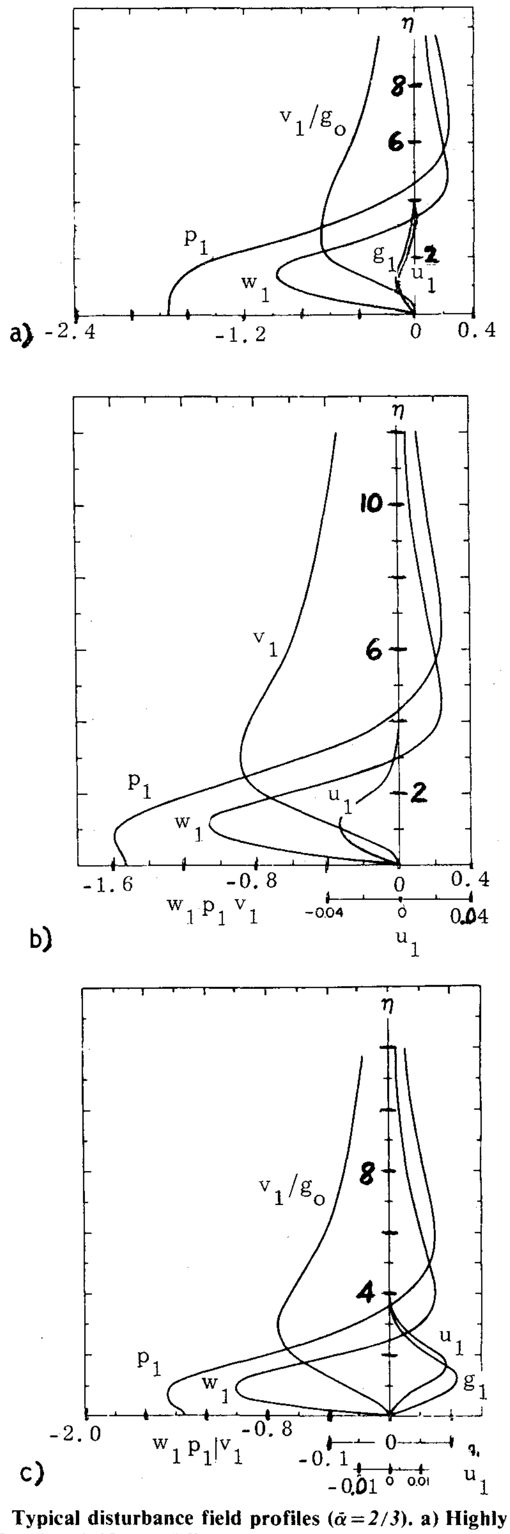


**Fig. 4 Typical disturbance field profiles ($\bar{\alpha}=2/3$). a) Highly cooled wall ($T_w/T_0=0.15$). b) Adiabatic wall ($T_w=T_0$). c) Hot wall ($T_w/T_0=2.0$).**

Some representative sets of disturbance field profiles across the reattachment region are shown in Figs. 4 for various degrees of wall cooling (hot wall, adiabatic, and cold wall surface temperatures); the corresponding zeroth-order solution profiles for $f_0'$ and $g_0$ also are available.[13] In accord with assumption 1) in Sec. IIA, there has been *no* arbitrary amplitude normalization of these perturbation solutions; all results plotted and tabulated here come directly from Eqs. (4-8). It is seen that the spanwise disturbance velocity changes sign around $\eta \simeq 3.4$ (slightly outside the basic reattachment flow boundary layer), indicating the vortex core at this level.

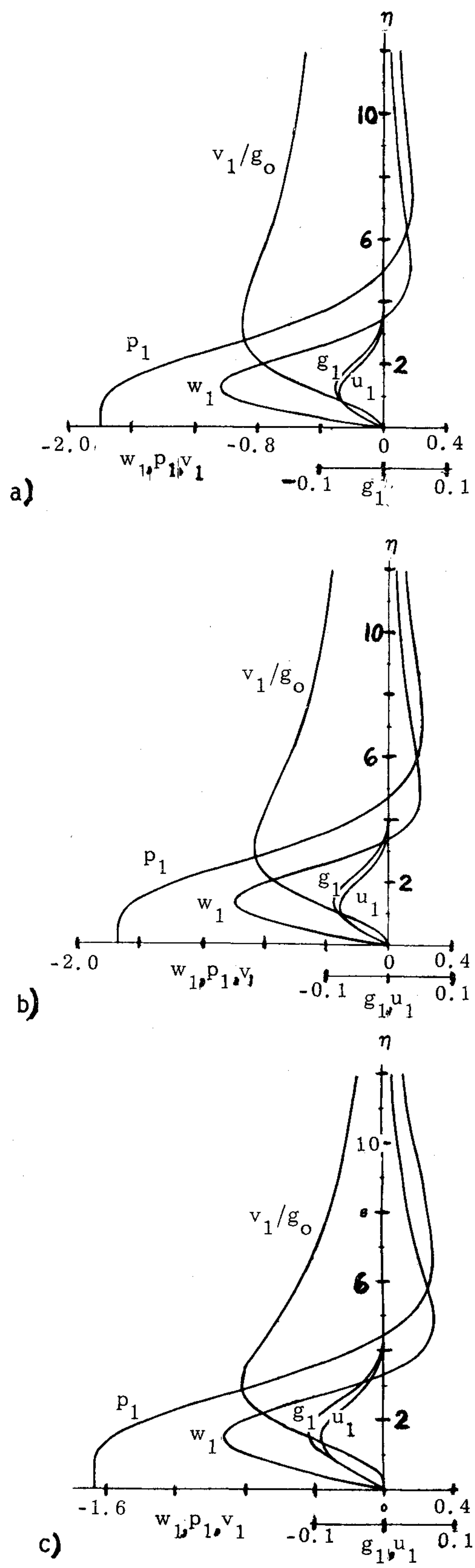


**Fig. 5 Mass-transfer effect disturbance profiles ($T_w=0.15\ T_0$, $\bar{\alpha}=2/3$) a) Suction, $f_{0w}=0.30$. b) Suction, $f_{02}=0.10$. c) Suction, $f_{0w}=0.10$.**

The effects of viscosity and heat conduction on the disturbance field become important about halfway into the basic boundary layer ($y \le 1/2\delta_0$) and serve to damp out the perturbations. A significant lateral pressure drop occurs, mostly in the inviscid disturbance region.

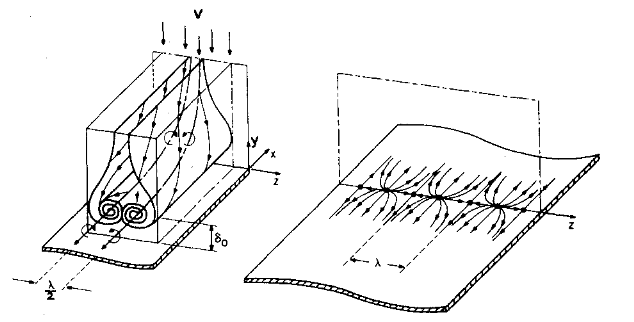

Fig. 6 Disturbance vortex flowfield (schematic).

The influence of surface mass transfer is shown in Fig. 5. As expected, the disturbance heat transfer and skin friction both decrease slightly with increased blowing, while suction not only increases these, but also the spanwise pressure perturbations. The significant mass-transfer effect on the far-field behavior of $v_1$ perceived in Fig. 5 is due to the sensitivity of the asymptotic solution to the value of $f_0$ and hence to the surface mass-transfer displacement effect $[v_1 \sim f_0^{-1} \sim (\eta - K_0)^{-1}$ where $K_0$ depends on $f_{0w}]$.

The corresponding disturbance motion involves pairs of counter-rotating vortices, which develop along the turning basic flow streamlines (see Fig. 6), and which become parallel to the surface outside the boundary layer. The spanwise disturbance streamline pattern very near the surface also is sketched in Fig. 6 and involves an alternating node and saddle-point structure along the mean reattachment line in agreement with experimental observation (Fig. 1).

Practical engineering relations to predict the wall pressure, shear stress, and heat transfer are directly forthcoming from the present analysis. For example, the spanwise pressure variation along the reattachment line is

$$p'_w(z) \simeq p_{0_s} P_1(0) \cos(\bar{\alpha} z \sqrt{B/\nu_{0_s}}) \qquad (19)$$

where values of $P_1(0)$ are tabulated in Tables 1 (heat-transfer effect) and 2 (mass-transfer effect). This pressure fluctuation has wavelength on the order $\sqrt{\nu/B} \sim \delta_0$. Since $P_1(0) \sim 0$, at $\alpha z = 0$ and $\pi$, the stagnation pressure is decreased and increased, respectively, corresponding to the spanwise stations of maximum vortex-induced upwash $(v_1 > 0)$ and downwash. The relative heat-transfer disturbance for a fixed wall temperature can be expressed as

$$\Delta \dot{q}_w(z)/\dot{q}_{w_0} = (g'_1(0)/g'_0(0)) \cos(\bar{\alpha} z \sqrt{B/\nu_0 s}) \qquad (20)$$

and can be quite significant in practice since $g'_1(0)/g'_0 \approx .2 - .4$ for highly cooled walls (Table 1). It can be seen from this table that cooling also slightly increases and decreases the wall pressure and spanwise shear stress perturbations, respectively, while strongly enhancing both the heat transfer and streamwise shear disturbances. Since $g'_1(0)$, $P_1(0)$, and $w'_1(0)$ are negative, the heat transfer and wall pressure perturbations are in phase with each other, but 90° out of phase with the lateral shear stress disturbance, which maximizes at the spanwise vortex positions (Fig. 7). Consequently, Reynold's analogy is *not* applicable between these three-dimensional heat-transfer and skin-friction disturbances. The influence of blowing and suction on the disturbance heat transfer and skin friction is in the expected direction, but is rather small (Table 2); the most significant effect is the 10-12% enhancement of the wall pressure and sidewash perturbations due to suction.

### B. Comparisons with Experiment

Since the theory cannot yield a discrete wavelength eigenvalue, a semiempirical method must be used to determine $\bar{\alpha}$ as follows[10]: A number of previous studies[14-17] suggest that the most highly amplified disturbance corresponds to an eigenvalue around $\bar{\alpha} \approx 1$, and we have adopted this value as a first approximation. Then working back through the compressibility transformation to calculate the boundary thickness, one obtains the theoretical estimate $\lambda/\delta_0 \approx 2\pi(1.8 + 0.8 g_{0_w})$. This result agrees with experiment on three counts. First, the prediction that $\lambda/\delta_0$ is essentially Mach and Reynolds number independent is confirmed by experiments over a wide range of flow conditions. Second, the theoretical values of $\lambda/\delta_0$ range from 2.4 on an adiabatic wall to 3.5 on a very cold wall $(g_{0_w} \to 0)$, and these are in good agreement with the spread of values $2 \le \lambda/\delta_0 \le 4$ quoted as

Table 1 Vortex eigensolution values at wall: heat-transfer effect[a] $(f_{ow} = 0)$

| $\bar{\alpha}$ | $P_1(0)$ | $W'_1(0)$ | $g'_1(0)$ | $u'_1(0)$ |
|---|---|---|---|---|
| | $g_{o_w} = 0.15[g'_o(0) = 0.441]$ | | | |
| 0.20 | −2.314 | −2.129 | −0.017 | −0.018 |
| 0.50 | −2.051 | −1.465 | −0.068 | −0.071 |
| 2/3 | −1.675 | −1.139 | −0.092 | −0.097 |
| 1.00 | −1.355 | −0.883 | −0.158 | −0.167 |
| 1.19 | −1.212 | −0.781 | −0.197 | −0.207 |
| | $g_{o_w} = 0.50[g'_o(0) = 0.271]$ | | | |
| 0.20 | −2.137 | −2.206 | −0.009 | −0.011 |
| 0.50 | −1.984 | −1.860 | −0.045 | −0.058 |
| 2/3 | −1.607 | −1.490 | −0.063 | −0.082 |
| 1.00 | −1.238 | −1.140 | −0.107 | −0.139 |
| 1.09 | −1.158 | −1.066 | −0.119 | −0.154 |
| | $g_{o_w} = 100[g'_o(0) = 0]$ | | | |
| 0.20 | −2.114 | −2.712 | 0.0 | −0.007 |
| 0.50 | −1.867 | −2.310 | 0.0 | −0.035 |
| 2/3 | −1.538 | −1.875 | 0.0 | −0.050 |
| 1.00 | −1.114 | −1.358 | 0.0 | −0.080 |
| | $g_{o_w} = 2.0[g'_1(0) = -0.616]$ | | | |
| 0.20 | −2.062 | −3.656 | 0.020 | 0.0014 |
| 0.50 | −1.686 | −2.926 | 0.095 | 0.0057 |
| 2/3 | −1.343 | −2.332 | 0.133 | 0.0071 |
| 0.89 | −1.042 | −1.813 | 0.179 | 0.0079 |

[a]These values have been updated and supercede those given in Ref. 10.

Table 2 Mass-transfer effect on wall perturbations $(g_w = 0.15, \bar{\alpha} = 2/3)$

| | $f_{o_w}$ | $f''_{o_w}$ | $g_{o''_w}$ | $P_1(0)$ | $W'_1(0)$ | $g'_1(0)$ | $u'_1(0)$ |
|---|---|---|---|---|---|---|---|
| Blowing | −0.30 | 0.555 | 0.281 | −1.673 | −0.864 | −0.080 | −0.077 |
| | −0.20 | 0.609 | 0.331 | −1.664 | −0.949 | −0.085 | −0.084 |
| | −0.10 | 0.675 | 0.385 | −1.665 | −1.040 | −0.089 | −0.091 |
| | 0 | 0.744 | 0.441 | −1.675 | −1.139 | −0.092 | −0.097 |
| Suction | 0.10 | 0.813 | 0.499 | −1.731 | −1.275 | −0.098 | −0.106 |
| | 0.20 | 0.885 | 0.559 | −1.758 | −1.394 | −0.101 | −0.112 |
| | 0.30 | 0.958 | 0.622 | −1.795 | −1.526 | −0.104 | −0.118 |

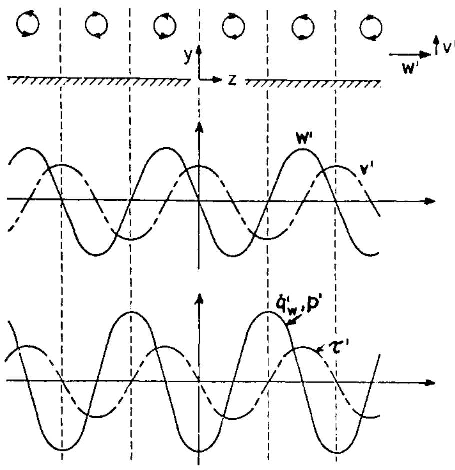


Fig. 7 Spanwise phasing of disturbance flow, surface pressure and heat transfer.

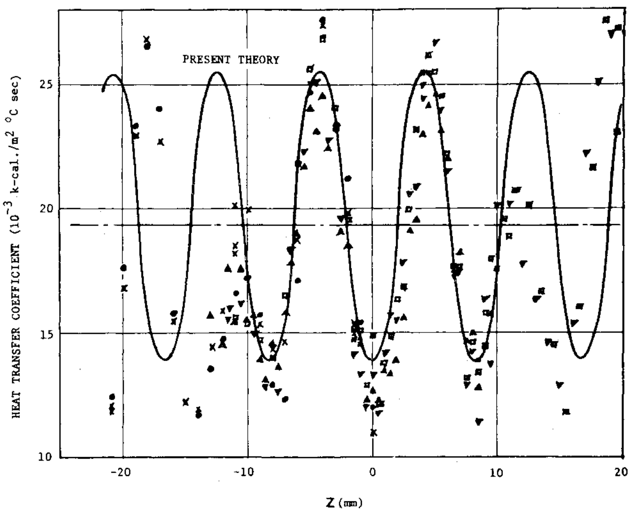


Fig. 8 Comparison of predicted spanwise heat-transfer perturbation with experimental data of Ginoux ($M \simeq 2.25$, $T_w/T_0 \simeq 0.30$).

being observed in *all* of the reported experimental studies.[1-9] Third, examination of the $v_1$ and $w_1$ velocity disturbance profile solutions for $\bar{\alpha}=1$ shows rough agreement in both the value and location of the maxima with available hot-wire study data of these reattachment disturbances.[17]

Using this wavelength selection without further adjustment, we have shown previously[10] that the present theoretical prediction for the spanwise static pressure variations agreed well (in both amplitude and wavelength) with Ginoux's detailed measurements.[11] Proceeding on the same basis (and again, with no "adjustment" of the amplitudes) Fig. 8 compares the theoretical heat-transfer perturbation from Eq. (20) with measurements[4] in the reattachment region following a two-dimensional rearward-facing step on a precooled model with $T_w/T_0 \simeq 0.30$. It is seen that the predictions are in excellent agreement with the experimental data. The laterally periodic heat-transfer variations are seen to be very pronounced indeed, having an amplitude of almost 40% about the mean in this example. It is felt that this good agreement with detailed heat-transfer measurements, together with the aforementioned experimental corroborations, validates the basic assumptions underlying our theoretical approach.

The typical thermal surface response to the vortex disturbance is illustrated in Fig. 9; this shows the spanwise heat-transfer pattern downstream of a rearward-facing step in supersonic flow, as observed by a sublimating surface film technique.[4] Another example is given in Fig. 10, where the

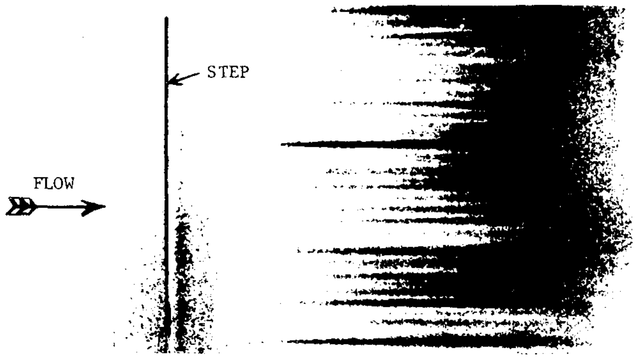


Fig. 9 Typical surface thermal response (subliming film[4]).

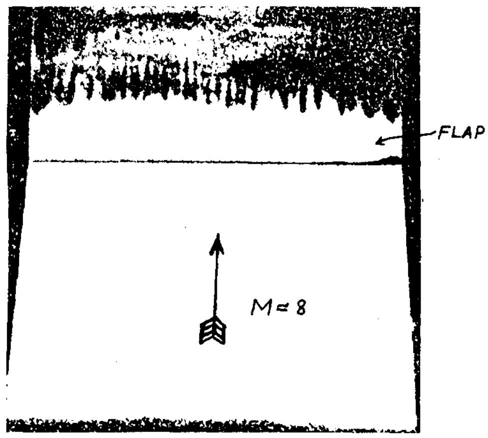


Fig. 10 Spanwise scorching pattern in hypersonic reattachment region (steel flap[3]).

scorch patterns observed in the reattachment region of a flap-induced separating $M=8$ flow[3] are shown. These patterns were observed by means of a temperature-sensitive paint coating with a low thermal conductivity surface material backing, and illustrate (in qualitative agreement with the suggestions of the present study) that spanwise-periodic disturbances can have significant effect on reattachment heat transfer in high-speed flows. It is noted that Miller et al.[3] also observed similar scorching patterns on highly-conductive pure stainless-steel flap model surfaces during hotshot wind-tunnel hypersonic flow tests, again qualitively corroborating the present surface response theory.

## IV. Concluding Remarks

The encouraging results of the present investigation notwithstanding, it is desirable to comment briefly on the limitations of the theory involved. First, by construction it does not deal with the effects of prescribed incoming disturbances, such as random freestream turbulence or various types of discrete vortices, since these were deemed unessential to the local Görtler-type mechanism of interest here. The effect of such sufficiently weak forced disturbances presumably could be superimposed on the present solutions. Second, the theory is limited to linearized small disturbances (although exactly how small is uncertain) and hence cannot capture any significant nonlinear effects. To be sure, it does seem to give an accurate account of the spanwise disturbance flow pattern and heat-transfer variations that have been observed across high-speed reattaching flows,[4,17] but further study of possible nonlinear coupling effects with small amounts of freestream turbulence and how these might influence the disturbance amplitudes and wavelength is desirable. Third, the present solution is restricted to the immediate neighborhood of the reattachment line; to study the vortex system development and heat-transfer downstream will require a significant extension of the mathematical analysis to include $0(x^2)$ and higher terms, plus possible basic boundary-layer transition to turbulent flow.

We conclude with some suggested further applications and extensions of the present work. 1) Although the primary focus here has been on the high-speed case, owing to the detailed experimental results available, the theory appears relevant also to the laminar stagnation line region of a cylinder in low-speed flow (indeed, there may be even a closer match with our theoretical assumptions). Such application should be carried out and compared with other available theories. 2) In order to extend it to hypersonic flow, the present two-dimensional reattachment flow model must be improved by including the effects of incoming flow vorticity and the adverse pressure gradient induced by viscous-inviscid interaction. Reference 10 suggests that the latter may influence the three-dimensional disturbances significantly. In this connection, there is a definite need for additional experimental data at hypersonic speeds ($M_\infty \simeq 4-10$) to establish the effect of high Mach number on the magnitude and wavelength of spanwise heat-transfer and pressure disturbances at reattachment. Judging by the meager evidence available and the implications of the present study [Eq. (10)], these heat-transfer disturbances (which are already appreciable even at moderate supersonic speeds) increase considerably with Mach number. 3) The treatment of surface transfer should be extended to include the case in which it also experiences a perturbation as well as wall heat conduction, thereby describing the response of subliming wall material coatings or ablative response in a manner analogous to our earlier work on the cross-hatching problem.[18] This is straightforward, owing to the linearity of the present theory and the mass-transfer sensitivity effect plus wall response analysis.

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## References

[1]Ginoux, J., "Experimental Evidence of Three-Dimensional Perturbations in the Reattachment of a Two-Dimensional Laminar Boundary Layer at $M=2.05$," Von Karman Institute TN-1, Brussels, Belgium, 1958.

[2]Hopkins, E.J., Keating, S.J., and Bandettini, A., "Photographic Evidence of Streamwise Arrays of Vortices in Boundary Layer Flow," NASA TN D-328, Sept. 1960.

[3]Miller, D.S. et. al., "Mach 8 to 22 Studies of Flow Separations due to Deflected Control Surfaces," *AIAA Journal,* Vol. 2, Feb. 1964, pp. 312-321.

[4]Ginoux, J., "Streamwise Vortices in Laminar Flow," AGARDograph 97, 1965, Pt. II.

[5]Roshko, A. and Thomke, G.J., "Observations of Turbulent Reattachment Behind and Axi-symmetric Downstream Facing Step in Supersonic Flow," *AIAA Journal,* Vol. 4, June 1966, pp. 975-980.

[6]Sfeir, A., "Supersonic Flow Separation on a Backward Facing Step," Univ. of California (Berkeley), Aerospace Science Division Rept. A5-66-18, 1966.

[7]Persen, L.N., "Investigation of Streamwise Vortex Systems Generated in Certain Classes of Curved Flow," Air Force Aerospace Research Labs., Wright Patterson AFB, Ohio, Rept. ARL 68-0134, July 1968.

[8]Gartling, K.K., "Tests of Vortex Generators to Prevent Separation of Supersonic Flow," University of Texas Applied Research Lab., Austin, Texas, Rept. TR-70-44, 1970.

[9]Rockwell, D.O. and Owczarek, J.A., "An Experimental Investigation of Three-Dimensionality of Wall Jet Flows," ASME Paper 72-Flcs-7, March 1972.

[10]Inger, G.R., "Three Dimensional Disturbances in Two-Dimensional Reattaching Flows," Virginia Polytechnic Institute and State University, Blacksburg, Va., Rept. Aero-011, Aug. 1974, presented to the *NATO-AGARD Symposium on Flow Separation* (AGARD CP-168), May 1975.

[11]Tani, I., "Production of Longitudinal Vortices in the Boundary Layer Along a Concave Wall," *Journal of Geophysical Research,* Vol. 67, March 1962, pp. 3075-3080.

[12]Bearman, P.W., "Some measurements of the Distortion of Turbulence Approaching a Two-Dimensional Bluff Body," *Journal of Fluid Mechanics,* Vol. 53, Pt. 3, 1972, pp. 451-466.

[13]Inger, G.R., "Three Dimensional Heat and Mass Transfer Effects Across High Speed Reattaching Flows," Virginia Polytechnic Institute and State University, Blacksburg, Va., Rept. Aero-036, July 1975.

[14]Hämmerlin, G., "Zur Instabilitates Theorie der ebenen Staupunkt-strömung," 50 Jahre Grenzschichtforschung, Vieweg, Braunschweig, 1955, p. 315.

[15]Kestin, J. and R.T. Wood, "On the Stability of Two-Dimensional Stagnation Flow," *Journal of Fluid Mechanics,* Vol. 44, pt. 33, 1970, pp. 461-479.

[16]Weiss, R.F., "Stability of the Stagnation Region to Three-Dimensional Disturbances," Ph.D. Thesis, New York University, April 1963.

[17]Ginoux, J., "On Some Properties of Reattaching Laminar and Transitional High Speed Flows," Von Karman Institute TN-53, Brussels, Belgium, Sept. 1969.

[18]Inger, G.R., "Three Dimensional Heat Transfer and Ablation Disturbances in High Speed Flows," *AIAA Journal,* Vol. 10, Dec. 1972, pp. 1641-1646.